

Answer Key:**Finding arc seconds per radian:**

$$(60 \text{ arc sec} / 1 \text{ arc min})(60 \text{ arc min} / 1 \text{ deg})(360 \text{ deg} / 2\pi \text{ rad}) = 206,265 \text{ arc sec/rad}$$

Finding the radius of Cas A in radians:

$$337 \text{ pixels} (0.5 \text{ arc sec/pixel}) (1 \text{ rad} / 206,265 \text{ arc sec}) = 0.000817 \text{ rad}$$

Converting the distance to Cas A to meters:

$$(11,100 \text{ light years})(9.46 \times 10^{15} \text{ meters} / 1 \text{ light year}) = 1.05 \times 10^{20} \text{ m}$$

Finding the radius of Cas A in meters:

$$\theta = (\text{radius of Cas A}) / (\text{distance to Cas A})$$

$$\text{radius of Cas A} = (\theta)(\text{distance to Cas A})$$

$$\text{radius of Cas A} = (0.000817 \text{ rad})(1.05 \times 10^{20} \text{ m}) = 8.58 \times 10^{16} \text{ m}$$

Finding the mass of gas contained in Cas A:

$$\text{Density} = \text{mass}/\text{volume}$$

$$\text{Volume} = 4/3 \pi r^3$$

$$\text{Mass} = (\text{density})(\text{volume}) = (10^{-21} \text{ kg/m}^3)(4/3 \pi)(8.58 \times 10^{16} \text{ m})^3 = 2.65 \times 10^{30} \text{ kg}$$

Finding the expansion velocity of Cas A:

$$\text{KE} = 1/2 mv^2 = 1/4 \times 10^{44} \text{ j}$$

$$V = \text{SQRT} (2 \text{ KE}/m) = \text{SQRT} [2(1/4 \times 10^{44} \text{ j}) / (2.65 \times 10^{30} \text{ kg})] = 4.35 \times 10^6 \text{ m/s}$$

Estimating the age of Cas A in seconds:

$$v = d/t$$

$$t = d/v = (8.58 \times 10^{16} \text{ m}) / (4.35 \times 10^6 \text{ m/s}) = 2.65 \times 10^{10} \text{ s}$$

Converting to years:

$$(2.65 \times 10^{10} \text{ s})(1 \text{ min} / 60 \text{ s})(1 \text{ h} / 60 \text{ min})(1 \text{ d} / 24 \text{ h}) (1 \text{ y} / 365.25 \text{ d}) = 625 \text{ y}$$

Displacement of the core remnant:

$$d = \text{sqrt} [(4292-4286)^2 + (4234-4252)^2] = 19 \text{ pixels}$$

$$19 \text{ pixels} (0.5 \text{ arc sec/pixel}) (1 \text{ rad} / 206,265 \text{ arc sec}) = 0.0000460 \text{ rad}$$

$$(0.0000460 \text{ rad})(1.05 \times 10^{20} \text{ m}) = 4.83 \times 10^{15} \text{ m}$$

Average velocity and kinetic energy of the core remnant:

$$v = d/t = (4.83 \times 10^{15} \text{ m}) / (2.65 \times 10^{10} \text{ s}) = 1.82 \times 10^5 \text{ m/s}$$

$$\text{KE} = 1/2 mv^2 = 1/2 (1.4)(2.0 \times 10^{30} \text{ kg})(1.82 \times 10^5 \text{ m/s})^2 = 4.6 \times 10^{40} \text{ j}$$

NOTE: John Flamsteed observed a supernova in the same location as Cas A approximately 330 years ago. 625 years has the same order of magnitude. With the assumptions and approximations made in this model, this could be the same supernova event, although there are other factors to be considered. Expansion probably does not occur at the same rate in all directions and the rate may not be constant over time.