Science Olympiad
Astronomy C
MIT Invitational

January 21, 2023

Directions:

- Each team will be given **50 minutes** to complete the test.
- There are three sections: **§A** (Multiple Choice) , **§B** (Free Response), and **§C** (JS9). Each section may have a mix of conceptual, quantitative, and DSO-related problems.
- Tiebreakers, in order: total from **§B**, total from **§C**, and then **§2** Question 2

Written by:
April Cheng, *aqc@mit.edu*;
Terry Matilsky, *matilsky@physics.rutgers.edu*;
Sahil Pontula, *spontula@mit.edu*

Section A: Multiple Choice

Use the images in Image Set A to answer the following questions. Each question is worth 1 point, for a total of 50 points.

1. List out the Harvard spectral classes in order of decreasing mass.
   (a) OBAFGKM
   (b) BFAMKGO
   (c) OGKMAFB
   (d) MKGFABO

2. List out the M-K luminosity classes in order of decreasing surface gravity.
   (a) 0, I, II, III, IV, V, VI, VII
   (b) I, II, VI, 0, V, IV, VII, III
   (c) III, 0, VI, I, IV, VII, V, II
   (d) VII, VI, V, IV, III, II, I, 0

3. List out the M-K luminosity classes in order of the stellar evolution of the sun.
   (a) I, II, III, IV
   (b) IV, V, VI, VII
   (c) V, IV, III, VII
   (d) VII, VI, V, I

4. True or False: more massive main sequence stars have higher temperature.
   (a) True
   (b) False

5. True or False: cooler main sequence stars tend to have less spectral lines.
   (a) True
   (b) False

6. True or False: $> 99\%$ of stars in the universe are main sequence stars.
   (a) True
   (b) False

7. True or False: the lower the mass, the more stars there are with that mass.
   (a) True
   (b) False

8. True or False: the lower the mass, the more stars we *observe* with that mass.
   (a) True
   (b) False

9. True or False: of all stars with mass 1 $M_\odot$, the largest stars tend to be the highest in temperature.
   (a) True
   (b) False

10. True or False: more massive white dwarfs are bigger in radius.
    (a) True
    (b) False

11. Type I supernovae are characterized by
    (a) The presence of Hydrogen in the spectrum
    (b) The absence of Hydrogen in the spectrum
    (c) The presence of Helium in the spectrum
    (d) The absence of Helium in the spectrum

12. Very massive stars die when fusion is no longer supported, causing a core-collapse supernova. Which of the following is NOT a core-collapse supernova?
    (a) Type Ia supernova
    (b) Type Ib supernova
    (c) Type II-P supernova
    (d) Type II-L supernova

13. The spectrum of a core-collapse supernova directly reflects
    (a) The temperature of the progenitor star
    (b) The mass of the progenitor star
    (c) The composition of the core of the progenitor star
    (d) The composition of the outer layer of the progenitor star

14. There is a cutoff mass at which stars will end their lives in core-collapse supernovae. This mass is closest to
    (a) 4 $M_\odot$
    (b) 8 $M_\odot$
    (c) 15 $M_\odot$
    (d) 20 $M_\odot$
    (e) There is no cutoff mass; whether or not a star will undergo a core-collapse supernova is strongly sensitive to other factors

15. Which of the following is NOT a possible remnant of a core-collapse supernova?
    (a) White dwarf
    (b) Neutron star
    (c) Black hole
    (d) Nothing (no remnant)
    (e) Both a and d

16. What kind of stars do we expect to see in a typical globular cluster in the Milky Way?
    (a) Population I stars
    (b) Population II stars
    (c) Population III stars
    (d) Both (b) and (c)
    (e) None of the above

17. Arrange pulsars, millisecond pulsars (MSPs), and magnetars in order from oldest to youngest. *(Here, pulsars refer to ordinary pulsars that are not magnetars or MSPs.)*
    (a) Pulsars, magnetars, MSPs
    (b) Pulsars, MSPs, magnetars
    (c) MSPs, pulsars, magnetars
    (d) Magnetars, pulsars, MSPs

18. Which of the following are rarely or never found in globular clusters?
    (a) Planetary nebulae
    (b) Luminous Blue Variables
    (c) Magnetars
    (d) Type II supernovae
    (e) All of the above

19. Intermediate-mass Black Holes (IMBHs) in the Milky Way would probably be formed via
    (a) Hierarchical merging in a globular cluster or nuclear star cluster
    (b) Direct collapse of an extremely massive star
    (c) Direct collapse of a massive primordial gas cloud
    (d) Accretion of mass onto dark matter seeds

20. What are standard candles?
    (a) Astrophysical objects which emit at a constant, known luminosity
    (b) Astrophysical objects whose luminosity can be calibrated via measurements independent of flux and distance
    (c) A standard unit of measuring luminosity in astronomy, such as $L_\odot$
    (d) Extremely luminous lasers on satellites in space to calibrate distances in the solar system

21. Which of the following is CANNOT be used as a standard candle?
    (a) Cepheid variables
    (b) RR Lyrae variables
    (c) Mira variables
    (d) Helium-flash red giant stars

22. What do dwarf novae, symbiotic stars, and x-ray binaries have in common?
    (a) The system is composed of a compact object and a non-compact donor star
    (b) Their outbursts are often caused by runaway fusion reactions on the surface of the compact object
    (c) They likely went through a common envelope phase
    (d) Both a and b
    (e) Both a and c

23. The fact that long Gamma-Ray Bursts are isotropically distributed across the sky is supporting evidence for which of the following facts?
    (a) Most of them are extragalactic
    (b) They're extremely intrinsically luminous
    (c) They're similar to supernovae in their astrophysical origin
    (d) Both (a) and (b)
    (e) both (a) and (c)

24. Suppose you plotted a histogram of the burst durations of a catalog of Gamma Ray Bursts. What would you see?
    (a) A unimodal, Gaussian-like distribution
    (b) A bimodal distribution
    (c) A left-skewed distribution
    (d) A right-skewed distribution

25. Why is it unusual to see carbon in a star's spectrum?
    (a) Carbon is not produced in stars
    (b) Only extremely massive stars produce carbon, which are less common
    (c) The wavelengths that carbon absorbs at are not in the visible spectrum
    (d) The fusion of helium into carbon occurs deep inside stars

26. Which of the following types of stars would we NOT expect to have carbon in their photospheres?
    (a) Mira variables
    (b) Carbon stars
    (c) Wolf-Rayet stars
    (d) O-type main sequence stars

27. Which of the following is true?
    (a) The Stefan-Boltzmann law can be derived from Planck's law
    (b) Planck's law can be derived from Wien's law
    (c) Wien's law can be derived from the Stefan-Boltzmann law
    (d) The Stefan-Boltzmann law can be derived from Wien's law

28. Model a star as a sphere filled of an ideal photon gas that emits according to the Stefan-Boltzmann law. How does the heat capacity of this gas scale with temperature?
    (a) T^{-3}
    (b) T^{-1}
    (c) T^{1}
    (d) T^{3}

For questions 29-35, refer to the image on the cover of your exam, which is a new image taken by the James Webb Space Telescope showing a binary star system. The components are unresolved in the center of the image. The rings surrounding the central system are dusty material ejected by one of the stars, periodically compressed by stellar winds when the binary system orbits close enough. (*Note: this is not a DSO on the event description. You do not need to know anything specific to the object besides what is given to answer the questions.*)

29. There is at least one star in the system that produces dust and stellar winds. Which of the following could this star be?
    (a) Pulsar
    (b) Classical Cepheid
    (c) Mira variable
    (d) Wolf-Rayet star

30. What can you infer about the orbit of the binary system?
    (a) It's highly eccentric
    (b) It has low eccentricity, or is near-circular
    (c) It's hyperbolic; the system will soon be unbound
    (d) There is not enough information to infer any of the above

31. Suppose that I've measured the wavelength of the $H - \alpha$ line of the system, which is broadly peaked and goes from 655.624 nm to 656.937 nm. Calculate the speed at which the rings are moving outward. (Note: I made up these numbers; they are not actual data on the system)
    (a) 3 km/s
    (b) 30 km/s
    (c) 300 km/s
    (d) 3000 km/s
32. Suppose that I've measured a parallax for the system of 0.58 mas. What's the distance to this system?
    (a) 100 pc
    (b) 500 pc
    (c) 1100 pc
    (d) 1700 pc
33. The angular distance between concentric rings in the image is 0.30". What is the physical distance between rings?
    (a) $7.6 \cdot 10^{10}$ km
    (b) $9.8 \cdot 10^{10}$ km
    (c) $3.2 \cdot 10^{11}$ km
    (d) $6.5 \cdot 10^{11}$ km
34. What's the orbital period of the system?
    (a) 4 years
    (b) 8 years
    (c) 13 years
    (d) 30 years
35. The total mass of the system is 30 $M_\odot$. What's the semi-major axis of the system?
    (a) 1 AU
    (b) 7 AU
    (c) 12 AU
    (d) 20 AU

Use the following information for the next three questions: A particle has zero total energy as it orbits a solar-mass star. The distance from the star at closest approach (periastron) is $d_p = 1$ AU.

36. What shape does its orbit trace?
    (a) Parabola
    (b) Hyperbola
    (c) Ellipse
    (d) Circle
37. What is the tangential component of its velocity in km/s at periastron?
    (a) 21 km/s
    (b) 42 km/s
    (c) 84 km/s
    (d) 100 km/s
38. What is the radial component of its velocity in km/s at periastron?
    (a) 0 km/s
    (b) 12 km/s
    (c) 24 km/s
    (d) 48 km/s

The remaining multiple-choice questions will be on Deep Sky Objects.

39. Identify the DSO in Image A.
    (a) AG Carinae
    (b) R Hydrae
    (c) R Aquarii
    (d) RS Puppis
    (e) W Virginis
40. What kind of object is this DSO?
    (a) Planetary nebula
    (b) H II region
    (c) Luminous Blue Variable
    (d) Dwarf nova
    (e) X-ray binary
41. This image includes data from which of the following wavelength regimes?
    (a) Near-UV
    (b) Visible light

(c) Near-infrared
(d) Both a and b
(e) a, b, and c

42. The surrounding ring of material was ejected around 10,000 years ago. Using the scale on the image, estimate the velocity at which the ring of material is travelling outwards.
(a) 1 km/s
(b) 10 km/s
(c) 100 km/s
(d) 1,000 km/s

43. Suppose the scale on the image was 6 light-years to 31", rather than 3 light-years to 31". Calculate the distance this DSO would have to be at for this scale to be accurate.
(a) 17 kly
(b) 25 kly
(c) 34 kly
(d) 40 kly

44. Identify the DSO in Image B.
(a) AG Carinae
(b) R Hydrae
(c) R Aquarii
(d) NGC 7027
(e) HD 184738

45. This object is much smaller than other objects of the same type. What does this suggest about this DSO?
(a) It is very inactive
(b) It is very old
(c) It is very young
(d) It suggests an unknown physical process at play

46. Identify the DSO in Image C. The left and middle images involve optical wavelengths, while the right image was taken in x-rays.
(a) NGC7027
(b) HD 184738
(c) 47 Tucanae
(d) E0102-72.3
(e) SN 2008D

47. The cross labels the location of the
(a) supernova remnant
(b) compact remnant
(c) central Wolf-Rayet star
(d) None of the above

48. Would you expect the optical ring (pink) to be concentric with the x-ray ring (right)?
(a) No, because they were emitted by different objects in the same system
(b) No, because one of the rings is a background object
(c) No, because they were emitted by different times
(d) Yes, because they should be emitted by the same object during the supernova

49. Assuming the optical and x-ray rings were emitted from the same object but at different times, which ring would you expect to have been emitted first?
(a) The X-ray ring, because it's farther from the star
(b) The X-ray ring, because it's at a higher temperature
(c) The optical ring, because it's closer to the star
(d) The optical ring, because it's at a lower temperature

50. As seen in the x-ray image, there is no extended x-ray emission besides the hot ring of material. This is suggests that
(a) The central compact object has little to no magnetic field
(b) The central compact object has a strong magnetic field
(c) The central compact object has an accretion disk
(d) The central compact object does not have an accretion disk

Section B: Free Response

Points are shown for each question or sub-question, for a total of 50 points. If showing work, please box your final answer.

1. [21 pts] For the following question, refer to images D, E, and F on the image sheet.
   (a) [1 pt] What is the name of the event shown in Image D?
   (b) [1 pt] What kind of signal was detected?
   (c) [2 pts] Briefly describe the astrophysical origins of this event.
   (d) [3 pts] Images E and F show posterior distributions of the component masses. One of them describes the event in Image A, while another describes a similar (but separate) event. Here, m_1 is the mass of the primary (more massive) component object. Briefly describe qualitatively how we can come up with this kind of figure with the data in Image D.
   (e) [3 pts] Using your answer from part (c), which figure (E or F) describes the event in Image D? Explain your answer.
   (f) [2 pts] The *chirp mass* of the event was measured to be $1.118^{+0.004}_{-0.002}\,\mathrm{M}_\odot$. What is chirp mass, and why can we measure it so much more precisely than the component masses?
   (g) [3 pts] As time passes and the event progresses, the frequency of the signal (increases / decreases) until it suddenly dies out altogether. Why should we expect this?
   (h) [1 pt] There seems to be no signal detected on the bottom panel. Why is this?
   (i) [2 pts] This is not a spatial image of the sky. Despite this, how could you use the information from Image D to infer some information on where the event happened in the sky?
   (j) [2 pts] Describe why this event was especially significant in the context of multimessenger astronomy.
   (k) [1 pt] What does this event say about our model on gamma ray bursts?

2. [29 pts] You might have heard that electron degeneracy pressure gives a force that can resist gravitational collapse. In this problem, we'll examine how that works.
   (a) [2 pts] List two stages of stellar evolution in which electron degeneracy pressure is a dominant force counteracting gravity in some part of a solar-mass star ("solar mass" here refers to the main sequence mass).
   (b) [1 pt] List the quantum mechanical principle/theory responsible for electron degeneracy.
   (c) [3 pts] Electrons have wave-particle duality, meaning in some cases they can be thought of as behaving like particles, while in others they behave more as waves. The smaller (i.e. more "quantum") you get, the more the wave nature matters, so that's what we'll consider here. Let's first consider one dimension, so our "wavefunction" looks like $\psi(x,t) \sim \cos(kx - \omega t + \phi)$ where $k = 2\pi/\lambda$ is the wavevector (λ being the wavelength) and ω is the angular frequency of the wave (ϕ is just some arbitrary phase that we don't need to worry about for now). Suppose we consider electronic waves in 1D, confined to the line stretching from $x = 0$ to $x = L$. We assume periodic boundary conditions so that $\psi(x,t) = \psi(x+L,t)$. This "quantizes" the wave, meaning that its wavelength can't just take any value - it is now discretized. Find the spacing Δk between consecutive harmonics allowed by the boundary conditions in terms of L.
   (d) [3 pts] Use the de Broglie relationship between the wavelength and momentum of a particle, $p = h/\lambda$, to derive an equation for the energy of a mode E in terms of the wavevector k. You can use the reduced Planck constant $\hbar = h/(2\pi)$ and the electron mass m.

(e) [5 pts] Electrons fill up states up to an energy level known as the Fermi energy E_F. Assuming the electrons fill up all modes up to E_F, use your results to compute the total energy of the system. You can assume each mode can be occupied by at most two electrons due to spin degeneracy and that the wavevector k_F corresponding to the Fermi energy satisfies $k_F \gg \Delta k$. Express your answer using k_F rather than E_F.

(f) [3 pts] Great! We're almost there. Let's make it realistic by finding the total energy for three dimensions. To make our lives easier, we'll change the procedure slightly: find the total number of modes occupied up to the Fermi energy and compute the total energy by multiplying by E_F. This won't give us the right prefactor as we worked out for the 1D case but at least gives an upper bound on the total energy. Again, don't forget spin degeneracy. As a hint, you'll want to consider a big box of dimensions $L \times L \times L$ with periodic boundary conditions in all three directions. What shape should the isoenergy surface $E(k) = E_F$ subtend in k space? Again, express your answer using k_F rather than E_F.

(g) [2 pts] Write down a meaningful definition of pressure that you can calculate from your result and use it to write the electron degeneracy pressure P. Remember we want pressure to be intensive!

(h) [3 pts] Congrats! We've just calculated the electron degeneracy pressure in our model. Find the Fermi energy (in eV) corresponding to a "Fermi temperature" of 10^6 K and use this to find P in Pa.

(i) [2 pts] How would you expect electron degeneracy pressure to compare to pressures in hydrostatic equilibrium? From the perspective of stellar evolution, why is this the case?

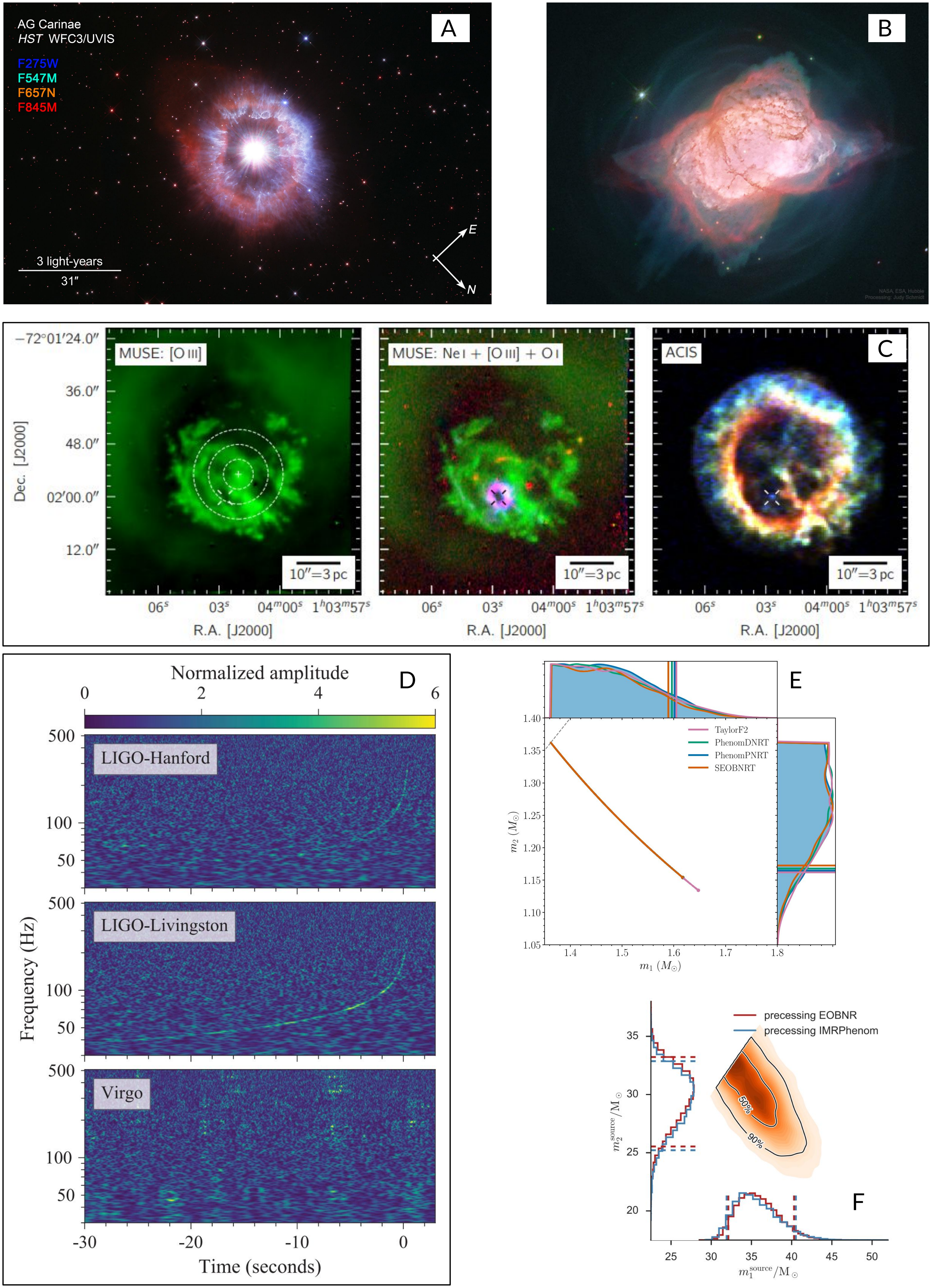
(j) [3 pts] List a stage of stellar evolution where neutron degeneracy plays an important role. Is the pressure stronger or weaker than electron degeneracy pressure? Explain why using your results above.

(k) [2 pts] The principle in (b) is what allowed us to do this problem and gives rise to the "degeneracy pressure." It does not apply to bosons (electrons and neutrons are fermions). Would you expect the boson pressure to be larger or smaller than the fermionic degeneracy pressure and why?

Section C: JS9

Points are shown for each question or sub-question, for a total of 10 points.

1. [10 pts] Use JS9 to analyze an observation of what has been called the most remarkable galaxy cluster in the Universe. Go to https://chandra.harvard.edu/js9/index.html. Enter "bullet cluster" as an object name in the Chandra archive. Hit search.
   (a) [1 pt] How many observations of the cluster has the Chandra satellite made?
   (b) [1 pt] What is the ObsID of the observation which will give you the greatest detail?
   (c) [1 pt] Why did you select this observation?
   (d) [1 pt] Load this observation into JS9 by dragging the "Title" to the JS9 window. Adjust the brightness/contrast so you can see the image most clearly. Do an energy spectrum under "analysis". In one or two sentences, describe the most prominent features of the data. Leave this energy spectrum in place.
   (e) [2 pts] Now, create a circular region, and drag it to encompass most of the cluster. Do another energy spectrum. What do you see that is different from your answer to d)?
   (f) [2 pts] What is/are possible explanation(s) of this difference?
   (g) [2 pts] What is the importance of the Bullet Cluster?



A
AG Carinae
HST WFC3/UVIS
F275W
F547M
F657N
F845M
3 light-years
31″
E
N
B
NASA, ESA, Hubble
Processing: Judy Schmidt
C
MUSE: [O III]
MUSE: Ne I + [O III] + O I
ACIS
10″=3 pc
−72°01′24.0″
36.0″
48.0″
02′00.0″
12.0″
Dec. [J2000]
06ˢ
03ˢ
04ᵐ00ˢ
1ʰ03ᵐ57ˢ
R.A. [J2000]
D
Normalized amplitude
0
2
4
6
LIGO-Hanford
LIGO-Livingston
Virgo
500
100
50
Frequency (Hz)
-30
-20
-10
0
Time (seconds)
E
TaylorF2
PhenomDNRT
PhenomPNRT
SEOBNRT
m2 (M⊙)
m1 (M⊙)
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.4
1.5
1.6
1.7
1.8
F
precessing EOBNR
precessing IMRPhenom
50%
90%
m2^source/M⊙
m1^source/M⊙
35
30
25
20
25
30
35
40
45
50

Section A (50 points)

1. ______	2. ______	3. ______	4. ______	5. ______
6. ______	7. ______	8. ______	9. ______	10. ______
11. ______	12. ______	13. ______	14. ______	15. ______
16. ______	17. ______	18. ______	19. ______	20. ______
21. ______	22. ______	23. ______	24. ______	25. ______
26. ______	27. ______	28. ______	29. ______	30. ______
31. ______	32. ______	33. ______	34. ______	35. ______
36. ______	37. ______	38. ______	39. ______	40. ______
41. ______	42. ______	43. ______	44. ______	45. ______
46. ______	47. ______	48. ______	49. ______	50. ______

Section B (50 points)

1. (a) ______
   (b) ______
   (c) ______
   (d) ______
   (e) ______
   (f) ______
   (g) ______
   (h) ______
   (i) ______
   (j) ______
   (k) ______
2. (a) ______

(b) ______________________________

(c) ______________________________

(d) ______________________________

(e)

(f)

(g)

(h)

(i) __________

(j) __________

(k) __________

Section C: JS9

1. (a) ______

   (b) ______

   (c) ______

   (d) ______

   (e) ______

   (f) ______

   ______

   (g) ______

   ______

Section A (50 points)

1. A	2. D	3. C	4. A	5. B
6. A	7. A	8. B	9. B	10. B
11. B	12. A	13. D	14. B	15. A
16. B	17. D	18. E	19. A	20. B
21. C	22. E	23. D	24. B	25. D
26. D	27. A	28. D	29. D	30. A
31. C	32. D	33. A	34. B	35. C
36. A	37. B	38. A	39. A	40. C
41. E	42. B	43. C	44. D	45. C
46. D	47. B	48. D	49. D	50. A

Section B (50 points)

1. (a) GW170817

   (b) Gravitational wave

   (c) **Merger of two neutron stars** in a binary orbit into a single black hole

   (d) Given the two masses, **we can predict the gravitational wave signal using general relativity** (1 point). We can use this to work backwards **using Bayesian analysis** (2 points) to construct a probability distribution for the masses.

   (e) **Figure E** (1 point), because the masses in Figure F are **too large to be neutron stars** (1 point).

   (f) The leading-order (i.e. most significant) term that contributes to the gravitational wave signal is determined by the chirp mass

   (g) Increases (1 point). The gravitational wave frequency is **directly proportional to the orbital frequency** (1 point), which **increases as the objects inspiral** (1 point).

   (h) The source was located in one of Virgo's blind spots

   (i) LIGO, KAGRA, and VIRGO are **located in different places on Earth** (1 point), so using **the differences in arrival time to each detector** (1 point), we can use triangulation to constrain the location of the source

   (j) This was the first and only simultaneous detection of **both gravitational and electromagnetic signals from the same event**.

   (k) Since a short-Gamma Ray Burst and a neutron star merger were found to be the same event, this event confirmed that **short-GRBs are kilonovae**.

2. (a) Red giant (core), white dwarf (1 point each).

   (b) Pauli exclusion principle (1 point).

   (c) The spacing is $\Delta k = 2\pi/L$ (3 points).

(d) $E = \frac{\hbar^2 k^2}{2m}$ (3 points).

(e) We compute

$$\begin{aligned} E_{\text{tot}} &= 2 \sum_{n=0}^{k_F/(2\pi/L)} \frac{\hbar^2(2\pi n/L)^2}{2m} \\ &\approx \frac{\hbar^2 k_F^3 L}{6\pi m} \end{aligned} \tag{1}$$

Full points should be awarded for a correct answer including prefactors (constants), 4 points for the correct combination of dimensional quantities, 3 points for the right scaling with k_F.

(f) Now, we have

$$\begin{aligned} E_{\text{tot}} &= 2 \frac{\frac{4}{3}\pi k_F^3}{(2\pi/L)^3} \frac{\hbar^2 k_F^2}{2m} \\ &= \frac{\hbar^2 k_F^5 L^3}{6\pi^2 m} \end{aligned} \tag{2}$$

Full points should be awarded for a correct answer including prefactors (constants), 2 points for the correct combination of dimensional quantities, 1 point for the right scaling with k_F.

(g) A reasonable definition is $P = E/V$ or $P = \partial E/\partial V$. It should look like $P = \frac{\hbar^2 k_F^5}{6\pi^2 m}$ modulo prefactors. Full credit for the combination $\frac{\hbar^2 k_F^5}{m}$.

(h) $E_F \approx 86$ eV (1.5 points) and $P \approx 4.4 \times 10^{13}$ Pa (1.5 points). Order of magnitude for the pressure is fine since the prefactors might be a bit different in (g).

(i) Larger (1 point); the inadequacy of main sequence hydrostatic pressure caused the need for degeneracy pressure in the first place to counteract gravitational collapse. Pressure due to a classical fluid is not sufficient and when the star collapses to a neutron star or white dwarf, the densities are so high that degeneracy pressure, which is quantum mechanical, dominates (1 point).

(j) Neutron star (0.5 points). Stronger pressure (1.5 point). Assuming the same Fermi energy, $P \propto m^{3/2}$ (1 point).

(k) Smaller (1 point), more bosons can pack into low-energy states so they don't "repel" each other as much (1 point).

Section C: JS9

1. (a) 10

   (b) ObsID 5356

   (c) Longest exposure time

   (d) Two large maxima plus a broad continuum

   (e) The maximum at 7 KeV is gone

   (f) The 7 KeV emission could come from outside the region selected. Other explanations are also possible

   (g) It seems to indicate an indirect detection of dark matter separated from ordinary matter via gravitational lensing